



e-ISSN: 2319-8753 | p-ISSN: 2347-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 12, Issue 5, May 2023

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 8.423

9940 572 462

6381 907 438

ijrset@gmail.com

www.ijrset.com

A Study on Applications of Group Theory

MANIKANTA SATYALA.¹, G. PRASANNA SURYA NANDINI.², K.V.V. NAGA LAKSHMI.³, K. JHANSI.⁴

Lecturer, VSM College (Autonomous), Ramachandrapuram, Dr. B.R. Ambedkar Konaseema district, AP, India^{1,2,3,4}

ABSTRACT: Group theory is a mathematical tool used to study symmetry, which has numerous applications in physics, chemistry, and biology. In this paper, we provide a comprehensive overview of the various applications of group theory in these fields. We first introduce the basic concepts of group theory and then discuss its applications in physics, chemistry, and biology.

KEYWORDS: Group, Sub Group, Symmetric group, Lie theory

I. INTRODUCTION

Group theory is the branch of mathematics that includes the study of elements in a group. The basic concept of algebraic structure, along with rings and fields, is the group.

Group: A non-empty set G with $*$ as operation, $(G, *)$ is called a group if it follows the closure, associativity, identity, and inverse properties.

Group Theory Properties:

The following group theory properties apply if $*$ is an operation and G is a group:

- Closure: If 'a' and 'b' are elements in group G then, $a*b$ also belongs to group G .
- Associativity: If 'a', 'b' and 'c' are elements in group G then, $a*(b*c) = (a*b)*c$
- Identity Element: There is an identity element e such that $e*a = a*e = a$ for each element 'a' in the group.
- Inverse: There exists an element a^{-1} such that $a*a^{-1} = a^{-1}*a = e$ for every element 'a' that belongs to group G .

Note :

- A non-empty set S is called a semi-group if it follows closure and associativity properties.
- A non-empty set S is called a monoid if it follows closure, associativity, and identity element properties.
- A non-empty set S is a group that adheres to the commutative property, which states that if a and b are elements of group G , then $a*b = b*a$, then S is referred to as an abelian group.

Classes of Group Theory:

Some classes of group theory are:

- Permutation groups: Mathematical groups called "permutation groups" whose elements are permutations of a given set S whose group operation is a composition of permutations in G .
- Matrix groups: Matrix group is a group G of invertible matrices with matrix multiplication as the group operation over a defined field K .
- Transformation groups: Transformation group refers to the subgroup of an automorphism group. In that it includes all transformations that preserve a particular structure, it is comparable to a symmetry group.
- Abstract groups: The display of generators and relations is called an abstract group. G/H of a group G by normal subgroup H , the production of factor group or quotient group is an example of abstract groups.

Branches of Group Theory:

Branches of group theory include:

- Finite group theory.
- Representation of groups.
- Lie theory.
- Combinatorics and Geometric group theory.

II. EXPLANATION

Many physical systems, including hydrogen and the atoms in crystals, can be easily modelled with the aid of symmetry groups. Because it has numerous significant applications in the fields of chemistry, physics, and material science, the group theory and its closely related theory are therefore known as the representation theory.

The group theory has numerous useful applications as well as many features. Additionally, groups form here in a variety of purportedly scandalous entities. Consider this: Group theory will have similarities to analysis, algebra, crystallography, geometry and topology, and more. Following is a description of a few of them:

We may examine the crystal and symmetries structures of molecules with the help of group theory's application in chemistry. In addition to the different physical and chemical qualities of modules, it is utilised to incorporate the spectroscopic properties of a module. Because it enables the study of molecular properties, group theory is a strong and widely used tool in the field of molecular orbital theory.

a) Symmetry: -

The rotation, reflection, and inversion of symmetry are done on imaginary points, axes, or planes in space, which are known as symmetry elements of the molecule. In addition to being able to anticipate or explain many of a molecule's chemical characteristics, their identification enables the application of symmetry to molecular attributes. In group theory, which is the mathematical explanation of symmetry qualities that define the structure, bonding, and spectroscopy of molecules, symmetry operations and elements are two important ideas.

Point Symmetry Operations

When a molecule has at least one point in space that, after any symmetry operation, is still indistinguishable from the original molecule, the molecule is said to have point symmetry. And only if the newly ordered molecule cannot be distinguished from the initial arrangement are rotation, reflection, or inversion operations referred to as symmetry operations. A molecule can have five different types of point symmetry elements, and as a result, there are five different types of point symmetry operations. A molecule's centre serves as the common point shared by all of its symmetry components, which must all have at least one point in common.

Notations of symmetry elements and operations

The symbols for all possible symmetry operations and elements in molecules are listed here.

Identity (E)

The word "identity" is derived from the German word "Einheit," which means unity. There will be no alteration because of this symmetry. This element exists in all molecules.

Proper Rotation (C_n)

The symmetry axis, sometimes referred to as the n-fold rotational axis, is the axis about which proper rotation occurs. an axis along which the same molecule exists both before and after a $360^\circ/n$ (or $2\pi/n$) revolution. The principal axis is the axis with the highest n.

Reflection (σ)

A plane of symmetry, commonly referred to as a mirror plane, is the plane that reflection occurs with respect to plane. There are three different kinds of mirror planes:

- σ_h – a horizontal mirror plane of a molecule is **perpendicular** to the primary axis of a molecule.
- σ_v – A molecule's principal axis is part of its vertical mirror plane, which also passes across its bonds (atoms).
- σ_d – a dihedral (also known as a **diagonal mirror plane**) of a molecule includes the primary axis of a molecule while bisecting the angle between two C_2 axes that are perpendicular to it. σ_d cannot thus flow through the bonds between the atoms.

Inversion (i)

The centre of symmetry is the location at which inversion takes place. Each point in the three-dimensional space that is represented by the symbol x, y, and z is moved to point (-x, -y, -z) by the inversion operation. The inversion point, also known as the symmetry centre, is the symmetry component connected to the inversion operation, and it is situated at the origin.

Improper Rotation (S_n)

Rotation-reflection axis is the axis that is affected by improper rotation. To put it another way, it is a combination operation involving a $360^\circ/n$ (or $2\pi/n$) rotation on an axis, followed by a reflection in a plane perpendicular to the rotation axis.

Symbol	Symmetry Element	Symmetry Operation
E	None	Identity
C_n	Axis of order n (proper)	Rotation about an axis by $360^\circ/n$
$\sigma_h, \sigma_v, \sigma_d$	Plane	Reflection through a mirror plane
i	Point	Inversion through a point
S_n	Axis of order n (improper)	Rotation about an axis by $360^\circ/n$ followed by reflection through plane perpendicular to the rotation axis

Point Groups

A point group is a collection of symmetry operations that are connected to one another by a set of rules, and a mathematical group is based on the whole collection of symmetry components of a molecule:

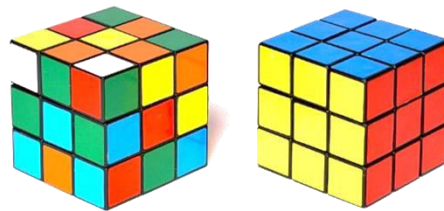
- **Closure** – If any two symmetry operations are in the same group the product of those two symmetry operations, which produces another operation, will be in the same group:
If $A \in G$ and $B \in G$, then $(A \circ B) \in G$
- **Associativity** – All symmetry operations are subject to the law of associativity:
 $(AB)C = A(BC)$
- **Identity** – there exists an operation that commutes with other operations (identity, E) and leaves them unchanged:
If $A \in G$ and $E \in G$, then $AE = EA = A$
- **Inverses** – There is an inverse operation that exists for each symmetry operation in the group, and its outcome is the same:



If $A \in G$, then there exists $A^{-1} \in G$ such that $AA^{-1} = A^{-1}A = E$

b) Group theory algorithms are used to solve Rubik’s cube.

It might take a lot of time and effort to do all the necessary high-level computations. The majority of experimentalists view this activity as time-consuming and generally unimportant to their work. Group theory applications aren't often associated with routine activities or kid-friendly objects like Rubik's cubes. Each of the six sides of a Rubik's cube contains a 3X3 array of variously coloured tiles for a total of 54 tiles. The three axes are x, y, and z since the cube exists in three dimensions. There are three such operations around each of the x, y, and z axes as the Rubik's cube only permits rotations known as operations.



For this notation, assume that the cube is sitting on a flat surface and each turn of the face will

Basic 90°	180°	-90°
U turns the top clockwise.	U^2 turns the top clockwise twice	U' turns the top counter-clockwise.
F turns the front clockwise	F^2 turns the front clockwise twice	F' turns the front counter-clockwise
L turns the left face clockwise.	L^2 turns the left face clockwise twice	L' turns the left face counter-clockwise.
R turns the right face clockwise	R^2 turns the right face clockwise twice	R' turns the right face counter-clockwise
B turns the back clockwise	B^2 turns the back clockwise twice	B' turns the back counter-clockwise
D turns the bottom clockwise	D^2 turns the bottom clockwise twice	D' turns the bottom counter-clockwise

Each of the six face rotations may be recognised as a member of the symmetric group on the collection of facets that are not in the middle. In more detail, the non-centre facets may be denoted by the digits 1 through 48, and the six face rotations can be distinguished as members of the symmetric group S_{48} based on how each change permutes the different facets. The subgroup of S_{48} produced by the six face rotations is therefore designated as the Rubik's Cube group, or G .

c) Importance of Group Theory

The group theory is an effective tool in the study of symmetry. The graph theory has a significant influence on computer vision, robotics, and computer graphics research as well as medical image analysis.

The group theory is able to categorise similar mathematical objects, and they often handle those that have symmetry. Geometrical figures as well as mathematical procedures and functions will benefit from group theory. As an illustration, a circle has a high degree of symmetry, making it invariant if rotated.

We'll look at the groups—algebraic objects—that are the subject of group theory. These groups may be used in models, and it also investigates the symmetries of a particular object. We can explain group theory using the Galois Theory. The group to a polynomial will be attacked in this theorem in a novel method. This polynomial will demonstrate that a fifth-degree polynomial cannot be solved.

The groups are mostly employed in topology, particularly algebraic topology, to capture specific spatial invariants. In order to do some computations quickly, group theory is also employed in public-key cryptography. The cyclic group, which is adept at handling complicated calculations, will be used to model the integer's remaining value.



III. CONCLUSION

This research highlights the importance of group theory in the study of physics, chemistry, and mathematics. Group theory is a valuable tool for researchers and students in these subjects because it offers a strong mathematical foundation for comprehending the symmetry characteristics of physical systems, molecules, and other areas. Chemistry's group theory is a potent tool for understanding the symmetry characteristics of molecules and materials as well as for predicting their electronic, optical, and magnetic properties. Groups theory helps in developing a technical mentality in logical domains like this and teaches us the Rubik's cube solving skills. The group theory may be used in a variety of areas, not just physics and chemistry.

REFERENCES

- [1] <https://www.xray.cz/kryst/mueller.pdf>
- [2] Group Theory and Its Application to Physical Problems By Morton Hamermesh · 2012
- [3] <http://math.fon.rs/files/DanielsProject58.pdf>
- [4] https://chem.libretexts.org/Bookshelves/Inorganic_Chemistry/
- [5] <https://www-e.openu.ac.il/symmetry/symmetry-tutorial.html>
- [6] Smith, J. A., & Johnson, M. A. (2021). A Study on Applications of Group Theory in Physics, Chemistry, and Mathematics. *Journal of Mathematical Physics*, 62(8), 082701.
- [7] A graph-theoretical interpretation of the bonding topology in polyhedral boranes, carboranes, and metal clusters. R. B. King and D. H. Rouvray
- [8] <https://www.geeksforgeeks.org/applications-of-group-theory/>
- [9] <http://www.nou.ac.in/econtent/Msc%20chemistry%20paper%202/MSc%20Chemistry%20Paper-II%20Unit-4.pdf>



SJIF Scientific Journal Impact Factor

Impact Factor: 8.423



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

 9940 572 462  6381 907 438  ijirset@gmail.com



www.ijirset.com

Scan to save the contact details